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Multiple scattering approach to light scattering by arbitrarily shaped particles

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Abstract. A multiple scattering method is described for calculating extinction, absorption and scattering cross-sections for dielectric particles of arbitrary shape, whose dimensions are comparable to the wavelength of the incident radiation. Numerical application to spheres agrees well with the Mie theory. An extension for a collection of arbitrarily shaped particles with application to zodiacal light is also given.

1. Introduction

The Mie theory, which yields us exact solutions to the scattering of light by spherical particles (Mie 1908) has so far been generalised only to infinite cylinders (Kerker 1969) and spheroidal particles (Asano and Yamamoto 1975). An interesting approach to the light scattering by non-spherical particles is due to Purcell and Pennypacker (1973). The particle which scatters is compared to an array of polarisable elements located on a cubic lattice. In their model absorption, extinction and scattering cross-sections are obtained by numerical solution of a system of linear equations satisfied by the complex amplitudes of the dipole moments.

In § 2, taking up again the Purcell model for a particle of given shape, we present a method giving the various cross-sections by a multiple scattering development of the complex vector amplitude of each dipole moment. This method, numerically applied to spheres, gives results in good agreement with the Mie theory.

In § 3 we show that the intensity scattered by a collection of arbitrarily shaped particles is related to the two-particle correlation function. In § 4 we give an application to zodiacal light.

2. Multiple scattering method for a particle of given shape

2.1 Iterative development

Following Purcell and Pennypacker (1973) we treat a dielectric particle as an aggregate of N polarisable elements mounted on a cubic lattice. Each dipole is characterised by a complex electric polarisability connected to the refractive index $m = n - ik$ by the Clausius–Mossoti relation (Van de Hulst 1957)

$$\alpha_e = \frac{3}{4\pi n} \frac{m^2 - 1}{m^2 + 2} \tag{1}$$

where n is the number of atoms per volume unit. Here we take $n = d_0^{-3}$, d_0 being the spacing of dipoles on the cubic lattice. To generate a form consists of putting N dipoles on the sites of the lattice Λ.

The complex vector amplitude $\mathbf{d}_i$ of the dipole moment of the ith entity is related to the electric field acting on the ith entity by

$$\boldsymbol{d}_i = \alpha_e \boldsymbol{E}_i. \tag{2}$$

$\boldsymbol{E}_i$ is composed of an external field,

$$\boldsymbol{E}_i^{\text{ext}}(\boldsymbol{x}_i, t) = E_0 \boldsymbol{e}_z \exp[\mathrm{i}(kx_i) - \omega_0 t)], \tag{3}$$

and the fields radiated by the other dipoles (Born and Wolf 1964),

$$\boldsymbol{E}_{\text{dip}}\left(\boldsymbol{r}_i, t - \frac{r_{ij}}{c}\right) = \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\exp(\mathrm{i}kr_{ij})}{r_{ij}^3}\left\{[(\boldsymbol{r}_{ij} \wedge \boldsymbol{d}_j) \wedge \boldsymbol{r}_{ij}]k^2 + \frac{1-\mathrm{i}kr_{ij}}{r_{ij}^2}[3(\boldsymbol{d}_j \cdot \boldsymbol{r}_{ij})\boldsymbol{r}_{ij} - r_{ij}^2 \boldsymbol{d}_j]\right\}, \tag{4}$$

where r_{ij} is the distance between the ith and jth dipoles. $\boldsymbol{E}_{\text{dip}}(\boldsymbol{r}_i, t - r_{ij}/c)$ can be written as

$$\boldsymbol{E}_{\text{dip}}(\boldsymbol{r}_i, t - r_{ij}/c) = \sum_{j\neq i} T(\boldsymbol{r}_{ij}) \cdot \boldsymbol{d}_j \tag{5}$$

where

$$T(\boldsymbol{r}) = \frac{\mathrm{e}^{ikr}}{r^3}[k^2 r^2(\mathbb{1} - \hat{r}\hat{r}) + (1 - \mathrm{i}kr)(3\hat{r}\hat{r} - \mathbb{1})], \tag{6}$$

$\mathbb{1}$ being the unit tensor and $\hat{r} = \boldsymbol{r}/|\boldsymbol{r}|$.

We introduce the multiple scattering development of $\boldsymbol{d}_i$,

$$\boldsymbol{d}_i(\boldsymbol{x}_i, t) = \alpha_e E_0 \boldsymbol{e}_z \exp[\mathrm{i}(kx_i - \omega_0 t)] + \alpha_e^2 \sum_{\substack{j=1 \\ j\neq i}}^{N} T(r_{ij}), \boldsymbol{e}_z E_0 \exp[\mathrm{i}(kx_j - \omega_0 t)]$$

$$+ \alpha_e^3 \sum_{\substack{j=1 \\ j\neq i}}^{N} \sum_{\substack{l=1 \\ l\neq j}}^{N} T(r_{ij}) T(r_{jl})\, \boldsymbol{e}_z E_0 \exp[\mathrm{i}(kx_l - \omega_0 t)\boldsymbol{e} + \dots. \tag{7}$$

We will now give the expressions for the absorption, extinction and scattering cross-sections and the expression for the differential scattering cross-section.

2.2 Expressions for the various cross-sections

The main energy absorbed by a dipole $\boldsymbol{d}$, per unit time, in a complex representation is given by (Landau and Lifshitz 1952)

$$Q_{\text{abs}} = \tfrac{1}{2}\,\mathrm{Re}(\boldsymbol{d}\; \mathrm{d}\boldsymbol{E}^*/\mathrm{d}t). \tag{8}$$

The absorption cross-section of the particle reads:

$$\sigma_{\text{abs}} = -4\pi \frac{\omega_0}{c} \frac{1}{|E_0|^2} \frac{\mathrm{Im}\,\alpha_e}{|\alpha_e|^2} \sum_{i=1}^{N} |\boldsymbol{d}_i|^2 \tag{9}$$

with $\boldsymbol{d}_i$ given by (7). c is the speed of light.

The differential cross-section is given by

$$\frac{d\sigma}{d\Omega} = (\boldsymbol{I}_d \cdot \boldsymbol{\eta}) r^2 \frac{8\pi}{c} \frac{1}{|E_0|^2} \tag{10}$$

where

$$\boldsymbol{I}_d = \tfrac{1}{2} \operatorname{Re}(\boldsymbol{E}_{\text{diff}} \wedge \boldsymbol{H}^*_{\text{diff}}), \tag{11}$$

$\boldsymbol{\eta}$ is the observation direction, and $\boldsymbol{E}_{\text{diff}}$ ($\boldsymbol{H}_{\text{diff}}$) is the electric (magnetic) scattered field obtained by summing the fields generated by the dipoles of the polarisable elements. Since the distance r separating the particle from the observer is large, we have

$$\boldsymbol{E}_{\text{diff}}(\boldsymbol{r}, t) = k^2 \frac{e^{ikr}}{r} \sum_{i=1}^{N} [\exp(-i\boldsymbol{k}_d \cdot \boldsymbol{x}_i)](\boldsymbol{\eta} \wedge \boldsymbol{d}_i) \wedge \boldsymbol{\eta} \tag{12}$$

and

$$\boldsymbol{H}_{\text{diff}}(\boldsymbol{r}, t) = k^2 \frac{e^{ikr}}{r} \sum_{i=1}^{N} [\exp(i\boldsymbol{k}_d \cdot \boldsymbol{x}_i)](\boldsymbol{d}_i \wedge \boldsymbol{\eta}) \tag{13}$$

with $\boldsymbol{k}_d = k\boldsymbol{\eta}$ and $\boldsymbol{d}_i$ given by (7).

From the optical theorem (Born and Wolf 1964) we deduce the extinction cross-section

$$\sigma_{\text{ext}} = -4\pi k \operatorname{Im}\left(\sum_{i=1}^{N} e^{ikx_i} d_{iz} \right) \tag{14}$$

where $d_{iz} = d_i(z, t)$.
The scattering cross-section can be obtained by integrating the expression (10) for the differential cross-section on the angular variables or through the difference

$$\sigma_{\text{diff}} = \sigma_{\text{ext}} - \sigma_{\text{abs}}. \tag{15}$$

At zero order we have

$$(\sigma_{\text{ext}})_0 = (\sigma_{\text{abs}})_0 = -4\pi(\omega_0/c) N \operatorname{Im} \alpha_e. \tag{16}$$

Putting $\boldsymbol{\eta} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, we obtain

$$\frac{d\sigma}{d\Omega} = \frac{4\pi}{c} k^4 |\alpha_e|^2 \left| \sum_{i=1}^{N} \exp(i\boldsymbol{k}_{d'} \cdot \boldsymbol{x}_i) \right|^2 \sin^2\theta \tag{17}$$

with $\boldsymbol{k}_{d'} = \boldsymbol{k} - \boldsymbol{k}_d$, which we identify with the Rayleigh scattering.

2.3 Numerical application

In order to calculate practically the expressions (9), (10), and (14) we introduce the recurrence formula

$$\boldsymbol{d}_i^{(n)} = \alpha_e^{n+1} \sum_{\{n\}} \mathscr{D}_i^{(n)} \tag{18}$$

where the summation means

$$\sum_{\{n\}} = \sum_{\substack{j_1=1 \\ j_1 \neq i}}^{N} \cdots \sum_{\substack{j_{n-1}=1 \\ j_{n-1} \neq j_{n-2}}}^{N} \tag{19}$$

and each component of $\mathscr{D}_i^{(n)}$ is calculated by the formulae

$$\mathscr{D}_{iz}^{(n)} = T_{31}^{(i,j_1)}\mathscr{D}_{ix}^{(n-1)}(j_1,\ldots,j_{n-1}) + T_{32}^{(i,j_1)}\mathscr{D}_{iy}^{(n-1)}(j_1,\ldots,j_{n-1}) + T_{33}^{(i,j_1)}\mathscr{D}_{iz}^{(n-1)}(j_1,\ldots,j_{n-1}) \quad (20)$$

$$\mathscr{D}_{iy}^{(n)} = T_{21}^{(i,j_1)}\mathscr{D}_{ix}^{(n-1)}(j_1,\ldots,j_{n-1}) + T_{22}^{(ij_1)}\mathscr{D}_{iy}^{(n-1)}(j,\ldots,j_{n-1}) + T_{23}^{(i,j_1)}\mathscr{D}_{iz}^{(n-1)}(j_1,\ldots,j_{n-1}) \quad (21)$$

$$\mathscr{D}_{ix}^{(n)} = T_{11}^{(i,j_1)}\mathscr{D}_{ix}^{(n-1)}(j_1,\ldots,j_{n-1}) + T_{21}^{(i,j_1)}\mathscr{D}_{iy}^{(n-1)}(j_1,\ldots,j_{n-1}) + T_{13}^{(i,j_1)}\mathscr{D}_{iz}^{(n-1)}(j_1,\ldots,j_{n-1}). \quad (22)$$

The matrix elements $T_{\alpha\beta}(\boldsymbol{r}_{ij}) \equiv T_{\alpha\beta}^{(i,j)}$ being defined in (6), the resulting differential cross-section takes the form:

$$\left(\frac{d\sigma}{d\Omega}\right)_n = \frac{4\pi}{c}k^4|\alpha_e|^{2(n+1)}\,\mathrm{Re}\left\{\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}[A]\exp[\mathrm{i}\boldsymbol{k}_d(\boldsymbol{x}_{i_1}-\boldsymbol{x}_{i_2})]\right\} \quad (23)$$

with

$$\begin{aligned}[A] = {}&\mathscr{D}_{i_1z}^{(n)}\mathscr{D}_{i_2z}^{*(n)}(1-\cos^2\theta) + \mathscr{D}_{i_1x}^{(n)}\mathscr{D}_{i_2x}^{*(n)}(1-\sin^2\theta\cos^2\phi)\\ &+ \mathscr{D}_{i_1y}^{(n)}\mathscr{D}_{i_2y}^{*(n)}(1-\sin^2\theta\sin^2\phi)\\ &-(\mathscr{D}_{i_1x}^{(n)}\mathscr{D}_{i_2y}^{*(n)} + \mathscr{D}_{i_1y}^{(n)}\mathscr{D}_{i_2x}^{*(n)})\sin^2\theta\sin\phi\cos\phi\\ &-(\mathscr{D}_{i_1x}^{(n)}\mathscr{D}_{i_2z}^{*(n)} + \mathscr{D}_{i_1z}^{(n)}\mathscr{D}_{i_2x}^{*(n)})\sin\theta\cos\theta\cos\phi\\ &-(\mathscr{D}_{i_1y}^{(n)}\mathscr{D}_{i_2z}^{*(n)} + \mathscr{D}_{i_1z}^{(n)}\mathscr{D}_{i_2x}^{*(n)})\sin\theta\cos\theta\sin\phi \end{aligned} \quad (24)$$

where

$\mathscr{D}_{i_1}^{(n)}$ is related to the index $i_1, j_{(1)_1}, \ldots, j_{(n-1)_1}$

$\mathscr{D}_{i_2}^{(n)}$ is related to the index $i_2, j_{(1)_2}, \ldots, j_{(n-1)_2}$.

In order to test the validity of the multiple scattering development we settled a computational method that we applied to spheres. For spheres of size parameters $ka = 1$ and $ka = 1{\cdot}5$ with a refractive index $m = 1{\cdot}7 - 0{\cdot}1\mathrm{i}$, which are approximated by a cluster of 248 polarisable entities, our results are in good agreement with the Mie theory to approximately 1%.

For $ka = 1{\cdot}5$, we obtain

$$\sigma_{\text{ext}} = 1{\cdot}951\,4179, \qquad \sigma_{\text{abs}} = 0{\cdot}599\,096\,67, \qquad \sigma_{\text{diff}} = 1{\cdot}352\,3212,$$

whereas the Mie theory yields

$$\sigma_{\text{ext}} = 1{\cdot}933\,148\,36, \qquad \sigma_{\text{abs}} = 0{\cdot}597\,818\,05, \qquad \sigma_{\text{diff}} = 1{\cdot}335\,330\,31.$$

These results have been obtained after twenty iterations. The multiple scattering series decreases very fast with n; for instance, after ten iterations we already get a good approximation to the final result. The computing time is around 1 mn on a CDC 7600. For a comparison, the Yung method (Yung 1978), which yields an accurate numerical treatment in solving the set of simultaneous equations for dipole amplitudes by means of a variational principle, requires the number of dipoles to be very large ($N = 4872$ for $x = 1{\cdot}5$) and a computing time around 10^{-6} N^2mn on a CDC 7600.

3. Extension to a collection of arbitrarily shaped particles

We search for the intensity scattered by a collection of arbitrarily shaped particles in a scattering volume V.

Taking the origin at the centre C of the particles, we have shown in § 1 that the scattered field is given by

$$\boldsymbol{E}_d(\boldsymbol{r}', t) = k^2 \frac{\mathrm{e}^{\mathrm{i}kr'}}{r'} \sum_{i=1}^{N} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_d \cdot \boldsymbol{x}_i} (\boldsymbol{\eta} \wedge \boldsymbol{d}_i) \wedge \boldsymbol{\eta}. \tag{25}$$

Now, if we take O as an origin, the scattered field becomes

$$\boldsymbol{E}_d(\boldsymbol{r}, t) = \sum_{i=1}^{N} k^2 \exp(-\mathrm{i}\boldsymbol{k}_d \cdot \boldsymbol{x}_i)[(\boldsymbol{\eta} \wedge \boldsymbol{d}_i) \wedge \boldsymbol{\eta}] \frac{\mathrm{e}^{\mathrm{i}kr}}{r} \exp(-\mathrm{i}\omega_0 t) \exp(\mathrm{i}\boldsymbol{K} \cdot \boldsymbol{r}_i) \tag{26}$$

with $\boldsymbol{K} = \boldsymbol{k} - k\boldsymbol{\eta}'$. r is the distance separating the origin O from the observer, $\boldsymbol{r}_i$ the distance OC, and $\boldsymbol{\eta}' = \boldsymbol{r}/|\boldsymbol{r}|$.
We put

$$\boldsymbol{E}_d(\boldsymbol{r}', t) = \boldsymbol{F}_f \frac{\mathrm{e}^{\mathrm{i}kr}}{r} \exp(-\mathrm{i}\omega_0 t) \exp(\mathrm{i}\boldsymbol{K} \cdot \boldsymbol{r}_i)$$

with

$$\boldsymbol{F}_f = \sum_{i=1}^{N} k^2 \exp(-\mathrm{i}\boldsymbol{k}_d \cdot \boldsymbol{x}_i)[(\boldsymbol{\eta} \wedge \boldsymbol{d}_i) \wedge \boldsymbol{\eta}]. \tag{27}$$

If $n_f(\boldsymbol{r}_i, t - r/c)$ is the density of particles of shape f the electric field scattered by a collection of particles is defined by

$$\mathscr{E}_d(\boldsymbol{r}, t) = \frac{\mathrm{e}^{\mathrm{i}kr}}{r} \mathrm{e}^{-\mathrm{i}\omega_0 t} \int_V \sum_f \boldsymbol{F}_f n_f\left(\boldsymbol{r}_i, t - \frac{r}{c}\right) \exp(\mathrm{i}\boldsymbol{K} \cdot \boldsymbol{r}_i) \, \mathrm{d}\boldsymbol{r}_i. \tag{28}$$

Taking the Fourier transform of $n(\boldsymbol{r}, t)$,

$$\tilde{n}(\boldsymbol{k}, t) = \int n(\boldsymbol{r}, t) \, \mathrm{e}^{\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}} \, \mathrm{d}^3\boldsymbol{r}, \tag{29}$$

we obtain

$$\mathscr{E}_d(\boldsymbol{r}, t) = \frac{\mathrm{e}^{\mathrm{i}kr}}{r} \mathrm{e}^{-\mathrm{i}\omega_0 t} \sum_f \boldsymbol{F}_f \tilde{n}_f\left(\boldsymbol{K}, t - \frac{r}{c}\right). \tag{30}$$

The scattered intensity calculated in the direction $\boldsymbol{\eta}'$ is given by the expression (Crosignani *et al* 1975)

$$I(\omega, \boldsymbol{\eta}) = \frac{c}{16\pi^2} \int_{-\infty}^{+\infty} \mathrm{d}\tau \, \mathrm{e}^{\mathrm{i}(\omega - \omega_0)\tau} \int_V \mathrm{d}\boldsymbol{r}_i \int_V \mathrm{d}\boldsymbol{r}_j$$

$$\times \sum_f \sum_{f'} \boldsymbol{F}_f \boldsymbol{F}_{f'}^* \exp[\mathrm{i}\boldsymbol{K} \cdot (\boldsymbol{r}_i - \boldsymbol{r}_j)] \langle n_f(\boldsymbol{r}_i, t - r/c + \tau) n_{f'}(\boldsymbol{r}_j, t - r/c) \rangle. \tag{31}$$

The mean value in (31) is related to the dynamical properties of the scattering particles, and will be discussed in the next section.

4. Application to zodiacal light

Following the work of Le Sergeant and Lamy (1978) on the interplanetary grains, the zodiacal particles can be divided into two populations: one consisting of particles whose dimensions are comparable to the incident wavelength and the other consisting of particles whose size is higher than the incident wavelength. For the first population $\boldsymbol{F}_f$ is well described by our multiple scattering approach. For the second population $\boldsymbol{F}_f$ can be satisfactorily described by an eikonal approach (Chiappetta 1980).

We will now calculate $\langle n_f(\boldsymbol{r}_i, t-r/c+\tau) n_{f'}(\boldsymbol{r}_j, t-r/c)\rangle$.

Two types of forces are acting on the zodiacal particles: (a) the gravitational force and the radiation pressure, and (b) the Lorentz force due to the electric and magnetic fields of the solar wind. We recall that $n_f(\boldsymbol{r}, t-r/c)$ is given by the Vlasov equation

$$\frac{\partial f}{\partial t}+\boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}}+\frac{\mu GM}{r^2}\frac{\partial f}{\partial v_r}+\frac{q}{m}[\boldsymbol{v}\wedge\boldsymbol{B}]\frac{\partial f}{\partial \boldsymbol{v}}+\frac{q}{m}\boldsymbol{E}\frac{\partial f}{\partial \boldsymbol{r}}=0 \tag{32}$$

where

$$n=\int f\,\mathrm{d}^3\boldsymbol{v}.$$

G is the gravitational constant, M the solar mass, q and m the particle charge and mass, and $\boldsymbol{E}$ ($\boldsymbol{B}$) the electric (magnetic) field in a frame moving with the solar wind.

According to Jokipii (1971) and Jokipii and Owens (1974) the magnetic field is:

$$\begin{aligned} &B_z=B_{\mathrm{P}}, \qquad B_x=\delta B_\theta, \\ &B_y=-\delta B_r\sin\psi-\delta B_\phi\cos\psi, \end{aligned} \tag{33}$$

where $\boldsymbol{B}_{\mathrm{P}}$ is the Parker field. We have $\boldsymbol{E}=-\boldsymbol{w}\wedge\boldsymbol{B}$ where $\boldsymbol{w}$ is the solar wind velocity. ψ is the angle between $\boldsymbol{r}$ and $\boldsymbol{B}$. The δB_i are the fluctuant components of the magnetic field. We seek for the solution of (33) in the form

$$f=f_0+f_1 \tag{34}$$

where f_0 is the solution corresponding to the Parker field and f_1 is the solution due to the fluctuations of the electric and magnetic fields.

We obtain

$$f_0(\boldsymbol{r},\boldsymbol{v})=C\exp\left\{-\frac{m}{2k_{\mathrm{B}}}\left[\frac{(v_\parallel-w_\parallel)^2}{T_\parallel}+\frac{v_\perp^2}{T_\perp}\right]\right\}\exp\left(\frac{\mu GMm}{k_{\mathrm{B}}T}\frac{1}{r}\right). \tag{35}$$

C is determined by the condition $n_0(\boldsymbol{r})=\int f_0(\boldsymbol{r},\boldsymbol{v})\,\mathrm{d}^3\,\boldsymbol{v}$ proportional to $r^{-\nu}$ with $1\leqslant\nu\leqslant 1{\cdot}3$ according to the different models. To the density $n_0(\boldsymbol{r})$ we must add the density due to fluctuations,

$$\delta n(\boldsymbol{K},\omega)=\int f_1(\boldsymbol{K},\boldsymbol{v},\omega)\,\mathrm{d}^3\boldsymbol{v}, \tag{36}$$

where f_1 is the solution of the equation

$$\frac{\partial f_1}{\partial t}+\boldsymbol{v}\frac{\partial f_1}{\partial \boldsymbol{r}}+\frac{\mu GM}{r^2}\frac{\partial f_1}{\partial v_r}+\frac{q}{m}[(\boldsymbol{v}-\boldsymbol{w})\wedge\boldsymbol{B}_{\mathrm{P}}]\frac{\partial f_1}{\partial \boldsymbol{v}}+\frac{q}{m}[\boldsymbol{v}\wedge\delta\boldsymbol{B}]\frac{\partial f_0}{\partial \boldsymbol{v}}+\frac{q}{m}\delta\boldsymbol{E}\frac{\partial f_0}{\partial \boldsymbol{v}}=0. \tag{37}$$

We solve (37) by observing that the Lorentz force can be treated perturbatively compared to the gravitational force. After a tedious calculation, the value $\langle|\delta\tilde{n}(\boldsymbol{K},\omega-$

$\omega_0)|^2\rangle$ related to $I(\omega, \boldsymbol{\eta}')$ is expressed by

$$\langle|\delta\tilde{n}(\boldsymbol{K}, \omega-\omega_0)|^2\rangle = \frac{2\pi^2}{\Omega_\parallel}C\left(\frac{k_B T_\perp}{m}\right)\sum_{l=-\infty}^{+\infty}\frac{\omega_c}{\Omega_c}\frac{1}{2\sqrt{\pi}}\exp\left[-\left(\frac{\omega-\omega_0-l\omega_c-K_\parallel w_\parallel}{\Omega_\parallel}\right)^2\right]$$
$$+\frac{q^2}{m^2}\left[\frac{C}{(2)^{3/4}}\frac{15\pi^3}{|\boldsymbol{K}|}\left(\frac{1}{\sqrt{2}}-1\right)\cos\frac{\pi}{8}\right]^2\left\langle\left|\sum_{l=-\infty}^{+\infty}\frac{\omega_c}{\Omega_\perp}\frac{1}{2\sqrt{\pi}}\left(\frac{T_\perp}{T_\parallel}\right)\right.\right.$$
$$\times\left(\frac{2\pi k_B T_\parallel}{mk_\parallel^2}\right)^{1/2}\delta_{E_\parallel}\,\mathrm{i}\sqrt{\pi}zl\,\mathrm{e}^{-zl^2}+\sum_{l=-\infty}^{+\infty}(\delta B_x+\delta B_y)\frac{l\omega_c^2}{k_\perp\Omega_\perp}$$
$$\times\left(1-\frac{T_\perp}{T_\parallel}\right)\left(\frac{2\pi k_B T_\parallel}{mk_\parallel^2}\right)^{1/2}\mathrm{i}\sqrt{\pi}zl\,\mathrm{e}^{-zl^2}+\sum_{l=-\infty}^{+\infty}\frac{(K_\perp/\omega_c)^{2l+1}}{2^{2l-1}\Gamma(l)}$$
$$\times\left(\frac{2k_B T_\perp}{m}\right)^{l-1/2}\frac{m}{k_B}\,{}_3F_3\left(l, l+\tfrac{1}{2}, l+1; l+1, l, 2l; -\frac{2k_B T_\perp K_\perp^2}{m\omega_c^2}\right)$$
$$\left.\left.\times\,\delta B_x\,\mathrm{i}\sqrt{\pi}zl\,\mathrm{e}^{-zl^2}\left(\frac{2\pi k_B T_\parallel}{mK_\parallel^2}\right)^{1/2}\left(\frac{1}{T_\parallel}-\frac{1}{T_\perp}\right)\right|^2\right\rangle \tag{38}$$

with

$$z_l=\left(\frac{m}{2k_B T_\parallel K_\parallel^2}\right)^{1/2}(\omega-\omega_0-l\omega_c-K_\parallel w_\parallel) \qquad w(z_l)=\frac{z_l}{\sqrt{\pi}}\int\frac{\mathrm{e}^{-x^2}\,\mathrm{d}x}{z_l-x}$$

$$\Omega_\perp^2=\frac{2k_B T_\perp K_\perp^2}{m} \qquad \Omega_\parallel^2=\frac{2k_B T_\parallel K_\parallel^2}{m}$$

and the scattered intensity is proportional to the Fourier transforms of the magnetic field correlation functions $\langle B_i(t)B_j(t+\tau)\rangle$.

From the expression (38) let us make two remarks. We can first evaluate the intensity width $\Delta\lambda$ due to fluctuations. For visible and ultraviolet spectra $l\omega_c$ and $k_\parallel w_\parallel$ are negligible. At a heliocentric distance $r = 1$au the zodiacal grains velocity is about 30 km s^{-1}. For an incident wavelength $\lambda_0 = 0{\cdot}5\mu$ and a scattering angle $\theta = \pi$ we obtain $\Delta\lambda = 0{\cdot}8$ Å.

The expression (38) can be separated into two terms. The first, which does not contain the correlation functions, is strongly peaked for $\omega = \omega_0$; the second term, proportional to the electric and magnetic field correlation functions, is maximal for $\omega - \omega_0 = \pm\Omega_\parallel$.

The expression (38) has been calculated above for a given incident wavelength and a heliocentric distance. Now in order to relate (38) to astronomical measurements we must integrate over all the solar spectrum wavelengths λ_0 and the line of sight.

If we perform the integration over λ_0, the first term of (38) will give a scattered spectrum very close to the solar incident spectrum.

Let us notice that to integrate the second term for a given scattered wavelength λ_s, we must take $-\Omega_\parallel$ and $+\Omega_\parallel$ as the limits of integration; a possible result would be a slight increase of scattered intensity in the ultraviolet domain.

5. Conclusion

The model which has been described allows us to calculate by a simple computational method the contribution of transition multipoles to the light scattering by a dielectric particle of irregular shape. This method can be generalised to particles containing impurities of one or several types and to inhomogeneous grains, each constituent or impurity corresponding to a different polarisability.

The model can also be generalised to particles having a non-negligible magnetic polarisability α_m connected to the magnetic permeability μ' by

$$\alpha_m = \frac{3}{4\pi n} \frac{\mu' - 1}{\mu' + 2}.$$

The complex vector amplitude of the magnetic moment of the ith entity $\boldsymbol{\mu}_i$ is related to the magnetic field acting on the ith entity by

$$\boldsymbol{\mu}_i = \alpha_m \boldsymbol{H}_i.$$

The magnetic field $\boldsymbol{H}_i$ is composed of the external field and the fields radiated by the other magnetic dipoles.

We mention that the polarisation can also be calculated using this method. In that case we take two incident fields,

$$E^{(1)}(\boldsymbol{r}, t) = E_0^{(1)} \boldsymbol{e}_z \, e^{i(kx - \omega_0 t)},$$

$$E^{(2)}(\boldsymbol{r}, t) = E_0^{(2)} \boldsymbol{e}_y \, e^{i(kx - \omega_0 t)}.$$

In conclusion, this multiple scattering approach can be applied to light scattering by a collection of arbitrarily shaped particles. The complexity of the scattering theory in this case is related to the statistical knowledge of the dynamical properties of the scattering particles.

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